

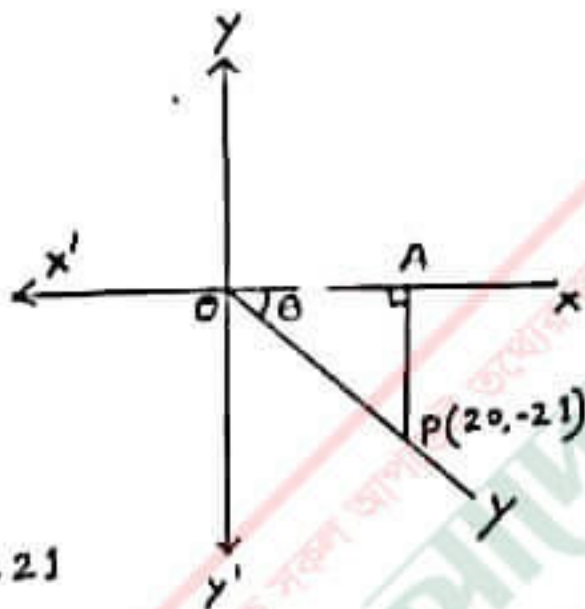
(ক)

$P(20, -21)$ বিন্দু
স্থূল Ox ও Oy উপর
 PA লম্বা অঙ্কন
করি।

তাহলে OAP একটি
সমকোণী-ত্রিভুজ।

$\triangle OAP - G,$

$$OA = 20, AP = -21$$



$$\begin{aligned} OP &= \sqrt{OA^2 + AP^2} \\ &= \sqrt{(20)^2 + (-21)^2} \\ &= \sqrt{400 + 441} \\ &= \sqrt{841} \\ &= 29 \end{aligned}$$

$\triangle OAP - G, \angle xOy = \angle AOP = \theta$

$$\cot \theta = \frac{OA}{AP} = \frac{20}{-21} = -\frac{20}{21}$$

$$\operatorname{cosec} \theta = \frac{OP}{AP} = \frac{29}{-21} = -\frac{29}{21}$$

$$\sin \theta = \frac{AP}{OP} = \frac{-21}{29}$$

$$\cos \theta = \frac{OA}{OP} = \frac{20}{29}$$

$$\text{শ্রুত রাশি} = \frac{\cot \theta + \operatorname{cosec} \theta}{\sin \theta + \cos \theta}$$

$$= \frac{-\frac{20}{21} - \frac{29}{21}}{-\frac{21}{29} + \frac{20}{29}}$$

$$= \frac{\frac{-20-29}{21}}{\frac{-21+20}{29}}$$

$$= -\frac{497}{213} \times \frac{29}{-1}$$

$$= \frac{203}{3} \quad (\text{Ans})$$

(খ)

$P(-\sqrt{3}, -1)$ বিন্দু হলে
OX এর উপর PA সম-
অক্ষ করি। তাহলে
 $\triangle OAP$ একটি সমকোণী
ত্রিভুজ।

$\triangle OAP$ -এ, $\angle AOP = \theta$

সে, $OA = -\sqrt{3}$, $AP = -1$

$$OP = \sqrt{OA^2 + AP^2}$$
$$= \sqrt{(-\sqrt{3})^2 + (-1)^2}$$

$$= \sqrt{3+1}$$

$$= \sqrt{4}$$

$$= 2$$

L.H.S

$$(Cot\theta - Cosec\theta + 1)(1 + \cos\theta)$$
$$= \left\{ \sqrt{3} - (-2) + 1 \right\} \left\{ 1 + \left(-\frac{\sqrt{3}}{2}\right) \right\}$$
$$= (\sqrt{3} + 2 + 1) \left(\frac{2 - \sqrt{3}}{2} \right)$$

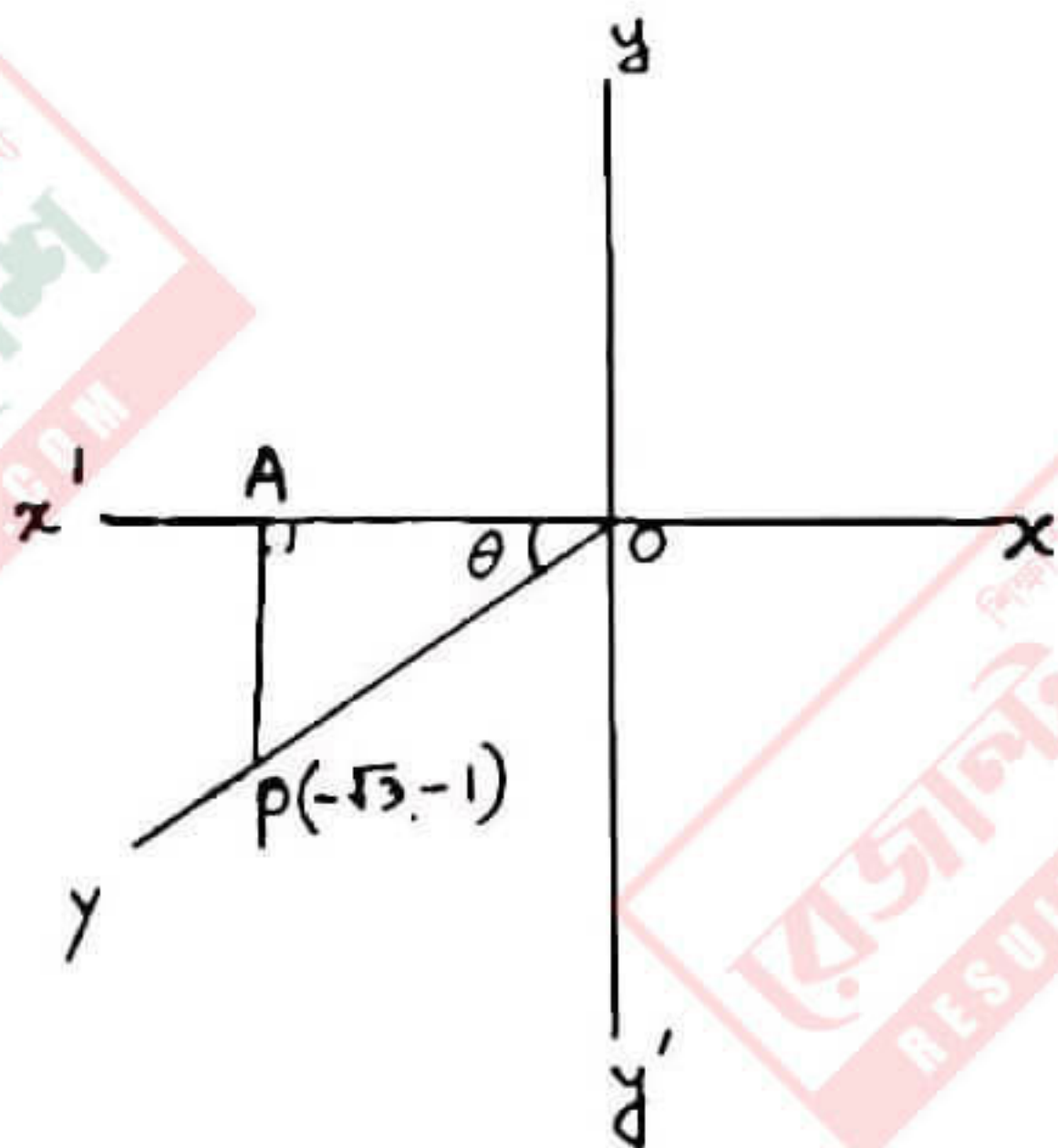
$$= \frac{(\sqrt{3} + 3)(2 - \sqrt{3})}{2}$$

$$= \frac{2\sqrt{3} - \sqrt{3}\sqrt{3} + 6 - 3\sqrt{3}}{2}$$

$$= \frac{-\sqrt{3} - 3 + 6}{2}$$

$$= \frac{3 - \sqrt{3}}{2}$$

\therefore L.H.S = H.S.S (shown)



$\triangle OAP$ সমকোণী ত্রিভুজ

$$Cot\theta = \frac{OA}{AP} = \frac{-\sqrt{3}}{-1} = \sqrt{3}$$

$$Cosec\theta = \frac{OP}{AP} = \frac{2}{-1} = -2$$

$$\cos\theta = \frac{OA}{OP} = \frac{-\sqrt{3}}{2}$$

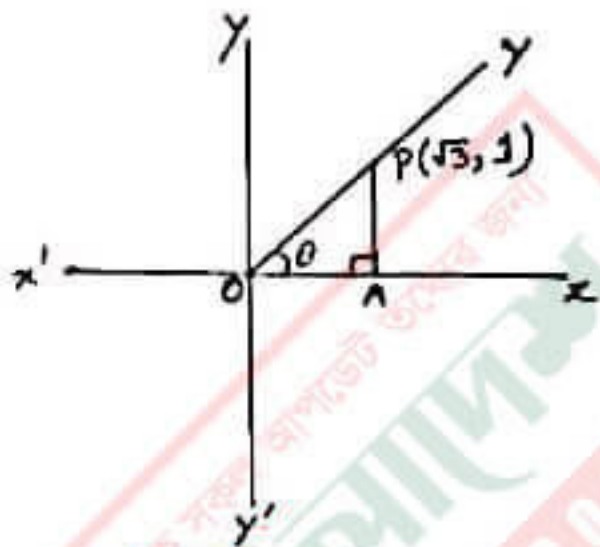
(১৭)

$P(\sqrt{3}, 1)$ বিন্দু হলে
OX বরাবর উল্লম্ব PA সমান্তরাল
যেহেতু। তাহলে $\triangle OPA$
একটি সামকোণী ত্রিভুজ

$\triangle OAP - G$

$$\angle AOP = \theta$$

$$OA = \sqrt{3} \quad AP = 1$$



$$OP = \sqrt{OA^2 + AP^2}$$
$$= \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$\triangle OAP - G$, $\sin \theta = \frac{AP}{OP}$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

সদস্য সমষ্টি = $\sum_{n=1}^6 \cos^2(2n-1)\theta$

$$= \cos^2(2 \times 1 - 1)\theta + \cos^2(2 \times 2 - 1)\theta + \cos^2(2 \times 3 - 1)\theta$$
$$+ \cos^2(2 \times 4 - 1)\theta + \cos^2(2 \times 5 - 1)\theta + \cos^2(2 \times 6 - 1)\theta$$

$$\begin{aligned}
 &= \cos^2 0^\circ + \cos^2 30^\circ + \cos^2 50^\circ + \cos^2 70^\circ + \cos^2 90^\circ \\
 &\quad + \cos^2 110^\circ \\
 &= \cos^2 30^\circ + \cos^2 9 \times 30^\circ + \cos^2 5 \times 30^\circ + \cos^2 7 \times 30^\circ + \cos^2 9 \times 30^\circ \\
 &\quad + \cos^2 11 \times 30^\circ \\
 &= \cos^2 30^\circ + \cos^2 90^\circ + \cos^2 150^\circ + \cos^2 210^\circ + \cos^2 270^\circ \\
 &\quad + \cos^2 330^\circ \\
 &= \cos^2 30^\circ + \cos^2 90^\circ + \cos^2 (180^\circ - 30^\circ) + \cos^2 (180^\circ + 30^\circ) \\
 &\quad + \cos^2 (3 \times 90^\circ + 0^\circ) + \cos^2 (4 \times 90^\circ + 90^\circ)
 \end{aligned}$$

$$= \cos^2 30^\circ + \cos^2 90^\circ + \cos^2 30^\circ + \cos^2 30^\circ + \sin^2 0^\circ + \cos^2 30^\circ$$

$$= 4 \times \cos^2 30^\circ + \cos^2 90^\circ + \sin^2 0^\circ$$

$$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^2 + 0^2 + 0^2$$

$$= 4 \times \frac{3}{4} + 0 + 0$$

$$= 3 \text{ (Ans.)}$$

(১৭) দেওয়া আছে,

$$\cos^4 \theta - \sin^4 \theta = 6\cos^2 \theta - 2(1 - 2\sin \theta); \quad 0 \leq \theta \leq 2\pi$$

$$\Rightarrow (\cos^2 \theta)^2 - (\sin^2 \theta)^2 = 6\cos^2 \theta - 2 + 4\sin \theta$$

$$\Rightarrow (\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) - 6\cos^2 \theta + 2 - 4\sin \theta = 0$$

$$\Rightarrow 1(\cos^2 \theta - \sin^2 \theta) - 6\cos^2 \theta + 2 - 4\sin \theta = 0$$

$$\Rightarrow \cos^2 \theta - \sin^2 \theta - 6\cos^2 \theta + 2 - 4\sin \theta = 0$$

$$[\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$\Rightarrow -5\cos^2 \theta - \sin^2 \theta + 2 - 4\sin \theta = 0$$

$$\Rightarrow -5(1 - \sin^2 \theta) - \sin^2 \theta + 2 - 4\sin \theta = 0$$

$$\Rightarrow -5 + 5\sin^2 \theta - \sin^2 \theta + 2 - 4\sin \theta = 0$$

$$\Rightarrow 4\sin^2 \theta - 4\sin \theta - 3 = 0$$

$$\Rightarrow 4\sin^2 \theta - 6\sin \theta + 2\sin \theta - 3 = 0$$

$$\Rightarrow 2\sin \theta (\sin \theta - 3) + 1(2\sin \theta - 3) = 0$$

$$\Rightarrow (2\sin \theta - 3)(2\sin \theta + 1) = 0$$

অথ, $2\sin \theta - 3 = 0$

$$\Rightarrow 2\sin \theta = 3$$

$$\Rightarrow \sin \theta = \frac{3}{2}$$

প্রকৃতপক্ষে অসম্ভব।

$$-1 \leq \sin \theta \leq 1$$

$$2\sin \theta + 1 = 0$$

$$\Rightarrow 2\sin \theta = -1$$

$$\Rightarrow \sin \theta = -\frac{1}{2}$$

$$\Rightarrow \sin \theta = -\sin \frac{\pi}{6}$$

আমরা জানি, $0 \leq \theta \leq 2\pi$ কক্ষীয় sine
এর মান ৩য় ও ৪র্থ চতুর্ভাগে ধনাত্মক।

প্রথম চতুর্ভাগে, $\sin \theta = \sin\left(\pi + \frac{\pi}{6}\right)$

$$\Rightarrow \sin \theta = \sin \frac{7\pi}{6}$$

$$\therefore \theta = \frac{7\pi}{6}$$

৪র্থ চতুর্ভাগে, $\sin \theta = \sin\left(2\pi - \frac{\pi}{6}\right)$

$$\Rightarrow \sin \theta = \sin \frac{12\pi - \pi}{6}$$

$$\Rightarrow \sin \theta = \sin \frac{11\pi}{6}$$

$$\therefore \theta = \frac{11\pi}{6}$$

\therefore নির্ণয় সমাধান, $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$.